Beamstrahlung at the Next Linear Collider

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Previous work on large angle incoherent beamstrahlung (IB) is extended to cover topics of relevance at the Next Linear Collider (NLC). These include feasibility at the NLC and the jitter monitoring capabilities of the method. The properties of coherent beamstrahlung (CB) in the microwave part of the spectrum (and its usage) are introduced. Some of its features are remarkable, and they include background-free detection, precision measurement of the beam-beam offset and beam length.

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I. INTRODUCTION.

There has been a lot of simulation and theoretical work about beamstrahlung at future linear colliders over the years, yet that work only scratches the surface of beamstrahlung phenomenology. Beamstrahlung is of interest to the particle physicist, who needs to know the energy distribution of colliding beam particles at collision time (dL/dE), and to the accelerator physicist who must make the beams collide and then steer the spent beams out of the Interaction Region.

In a series of recent papers we have made clear that recovering complete information on low energy beamstrahlung effectively recovers most of the available information about the beam-beam collision (BBC) in $e^+e^−$-colliders[1 – 3]. A large-angle infrared beamstrahlung detector is being built for CESR[3].

As remarked in Ref.[2], there are seven transverse degrees of freedom (dof) in the BBC(Fig. 1). These are a lot of dof and monitoring them is an unavoidable challenge. The technique discussed here is concerned with measuring as many of those dof as possible.

When the beams miss one another luminosity will decrease (or be wasted[2]). Beamstrahlung will also change (in most cases, it will increase) as one beam samples a different spatial region (with a different EM field) around the other beam, leading to a dL/dE curve which is both lower and wider. By monitoring beamstrahlung one recovers at once information about luminosity and its spectrum.

The idea underlying the usage of low energy beamstrahlung is actually a simple one. Given the Maxwell equations,

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$$  \hspace{1cm} (1)

the beams are the current $J$, and beamstrahlung is the emitted EM field $F$. The equations describe the correlation between currents and fields. We know the correlations, and we measure the field to figure out the currents. The field has vector properties, and that is why is necessary to measure their components. Measuring the polarization is crucial for this method. Fig. 2[2] shows four possible BBC, and fig. 3[2] shows the four corresponding beamstrahlung polarization components. Fig. 3 (called the beamstrahlung diagram) is a characterization of the solution of the Maxwell equations for the BBC.

In practice, it is difficult to measure the polarization of photons of energy higher than UV, and that is why we limit ourselves to the study of low energy beamstrahlung in the present paper. Coherent beamstrahlung is available for observation in the microwave region, however in that case the polarization information is not meaningful (see Section IV).

This paper is written with two goals in mind:

1. extend the previous work[1 – 3] to the NLC case.

Three topics need to be addressed: how much of the CESR concepts can be transferred to the NLC,
what is the forecast for signal and backgrounds, and most important of all, what can the method do about beam jitter (Section II);

2. introduce coherent beamstrahlung (CB), its properties and its potential (Section III).

II. INCOHERENT BEAMSTRAHLUNG AT THE NLC.

In view of the development of the coherent beamstrahlung technique in Section III, the technique developed in Refs. [1–3] and extended to the NLC in this Section is renamed incoherent beamstrahlung (IB). In Section III it will be noted that the distinction between IB and CB only depends on the wavelength at which the observation is made.

At the NLC beamstrahlung detection will be possible outside a 1 mrad cone, an angle that is still very large compared to 1/γ (1μrad). All properties of CESR IB that depend on the very large angle approximation will remain valid. Most important amongst them is the property that polarization information is retained at particular azimuthal locations [2, 5], even though the large angle radiation as a whole is unpolarized (Table I) (polarization is generally carried by high energy photons [4]).

At CESR, the large angle does two things for the experimenter: make the polarization observable, and separate signal and background. The detector has been placed at

![Diagram](image)

**FIG. 2:** a) the beams overlap perfectly, no luminosity is wasted; b) a y-offset; c) y-bloating; and d) a beam-beam rotation. The “bad” beam is represented by the dashed ellipse.

**FIG. 3:** Beamstrahlung diagrams corresponding to the four pathologies of Figure 2. The dashed vectors in parts a) and b) are slightly displaced for display purposes.

an angle of 10.4 mrad (or ~ 100/γ) [3], to be compared with the optical resolution of the device, 1.3 mrad, and the angular spread of machine backgrounds, a few mrad. At the NLC [7], a beamstrahlung power of order 1 MW imposes a stay-clear cone of 1 mrad (or ~ 1000/γ). Optically one can no longer hope to disentangle signal and background (assuming a diffraction-limited optical resolution of 1 mrad, as in the CESR case). Background will be rejected by other methods.

<table>
<thead>
<tr>
<th>Beam charge N</th>
<th>0.75 × 10^11 e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical beam width σ_y</td>
<td>3 mm</td>
</tr>
<tr>
<td>Horizontal beam width σ_x</td>
<td>243 mm</td>
</tr>
<tr>
<td>Beam length σ_z</td>
<td>110 μm</td>
</tr>
<tr>
<td>Beam energy</td>
<td>500 GeV</td>
</tr>
<tr>
<td>Beamstrahlung average energy loss</td>
<td>5.4%</td>
</tr>
<tr>
<td>Beamstrahlung (IB) yield, 350 &lt; λ &lt; 700 nm, 1 &lt; θ &lt; 2 mrad</td>
<td>3 × 10^8 photons</td>
</tr>
<tr>
<td>Beamstrahlung (IB) polarization 350 &lt; λ &lt; 700 nm, 1 &lt; θ &lt; 2 mrad</td>
<td>0</td>
</tr>
<tr>
<td>Beamstrahlung (CB) yield, offset = 3σ_y, 400 &lt; λ &lt; 500 μm, 1 &lt; θ &lt; 2 mrad</td>
<td>5 × 10^7</td>
</tr>
<tr>
<td>Beamstrahlung (CB) power, offset = 3σ_y, λ &gt; 100 μm</td>
<td>16 W</td>
</tr>
</tbody>
</table>

**TABLE I:** NLC nominal parameters and beamstrahlung yield for each bunch crossing [7].

IB power has no significant quantum corrections at low photon energy, making classical formulae precise enough
to be usable. The IB rates at the NLC for each bunch within the train are given in Table I, assuming full azimuthal acceptance. They are certainly abundant and capable of providing subpercent precision in the measurement of the beamstrahlung diagram. Because of jitter, it is particularly important that each BBC be recorded separately, and $10^8$ photons per BBC provide a viable starting point.

Background rejection will remain a major concern. If the signal is clearly extracted, separation of the polarization components is technically trivial. Three background rejection methods have already been suggested[8]. The first uses the fact that the beamstrahlung pulse is shorter than the coincident, synchrotron radiation (SR) background pulse by a factor of $2\sqrt{2}$. A streak chamber could disentangle the two components, with a possible background rejection of order $10^2$. A second method uses the fact that SR background tends to be strongly (90%) polarized radially[3]. By extracting only tangential components, one could reduce backgrounds by one order of magnitude with a signal loss of only a factor of two.

A much more powerful method than the previous two was the focus of Ref.[8]. An elliptical grating is the primary mirror, placed so that the Interaction Point (IP) is located at one of the ellipse foci, and the main collimator at the other focus. Such a device has extremely shallow field depth ($\sim 100\mu m$ at 10 meters distance). The background rejection is roughly equal to the number of lines in the grating ($\sim 10^4$). It has, however, also a very narrow frequency acceptance ($\sim 10^{-5}$), which may prove to be too large a signal reduction at the NLC.

Recently, I. Avrutsky[9] has systematically researched all possible methods of optical background rejection. He has found that whole-azimuth imaging, by a means of a ring-like mirror, offers at the same time a field depth of order one meter and a diffraction-limit of order 0.1 mrad. This method should allow background rejection at the $10^{-3}$ level, without any signal bandwidth loss. We conclude, preliminarily, that the signal will be abundant and that there exists the possibility to reduce backgrounds by several orders of magnitude if the need arises.

Finally, beam jitter is unique to the NLC and a potentially major technical problem. While nanometer-sized jitter is unimportant at present $e^+e^-$ colliders, the same effect at the NLC can seriously reduce luminosity. In the simplest possible case the beams could jump from the BBC of Fig. 2a to that of Fig. 2b and back. The corresponding diagram will jump from Fig 3a and Fig. 3b) and back. Bunch-to-bunch monitoring becomes a necessity, and Table I shows that the IB rates should be adequate for the task.

In Ref.[2], one assumption was made which is not true at the NLC. The beams were assumed to drift very slowly from one BBC to another. Implicit in the hypothesis was the fact that there was plenty of time for an observer to observe the drift, take corrective action by tuning a corrector magnet, and observe the new diagram obtained. Indeed the drift at CESR takes so long, that one can easily integrate the signal over one second (or 17 million BBC). That option will no longer be available at the NLC.

If beam motion is a pure drift, then Ref.[2] shows that the beamstrahlung diagram monitors effectively six of the seven $dof$ of Fig. 1, even though the diagram only has four dimensions. The extra two $dof$ can be had by applying a correction and observing a new diagram. The missing $dof$ is a scale factor equal to the smallest $\sigma_y$ of the two beams.

It is obvious that, jitter being a random process, control on the time evolution will be lost and there will be only four $dof$ available from the beamstrahlung diagram in the case of jitter. It becomes also imperative to record each and every bunch-bunch collision separately. If this can be done, then the diagram will provide passive information on the jitter amplitude and frequency in four dimensions, which is still far more information than most other methods can provide.

III. COHERENT BEAMSTRahlUNG AT THE NLC.

In the previous Section we have seen that IB loses some of its power at the NLC, due to jitter. We sought to find other beamstrahlung observables to restore the near-complete information that one can obtain at other machines. In the process we found that CB has remarkable properties which will be described in this Section.

The most remarkable property of CB is its very high power (Table I). One can see why by looking at the scaling laws of luminosity monitoring processes. For this purpose, any of several low-$Q^2$ QED processes can be used. For the purpose of discussion, let us consider $e^+e^- \rightarrow e^+e^-\gamma$. Most of these events (above a minimum angle) consist of one fermion and one photon at low angle in the same hemisphere, balanced in $p_T$, while the other fermion continues down the beam pipe. One can reasonably speak of a radiating beam, the beam in the same hemisphere as the photon. The event rate in each hemisphere, $R_{1,2}$, is proportional to the luminosity, and therefore to the product of the two beam populations

$$R_{1,2} \propto L \propto N_1 N_2.$$  \hfill (2)

When IB is considered, SR formulae are used. The power is proportional to the beam charge and proportional to the square of the bending force. This readily translates into a photon rate which goes like the cube of the current, and therefore a much larger yield:

$$R_1 \propto N_1 N_2^2.$$  \hfill (3)

When CB is used, the beam moves coherently under the influence of the EM field of the other beam. Radiation is proportional to the square of the emitting charge, so that

$$R_{1,2} \propto N_1^2 N_2^2.$$  \hfill (4)
A brilliant description of the coherent and incoherent limits for SR can be found in Ref.[10], which Concisely derives the $N_i$ and $N_i^2$ factors in Eqs. (3-4).

Equations (2-4) show at a glance why beamstrahlung is preferable to quantum processes - the $N$ factors are huge numbers which make for more abundant, more precisely measured rates. CB has other unique properties.

We have already noted at CESR that the overall $N^3$ dependence (Eq. 3) of IB does not favor the early development of the detector. Weak beams (a factor of ten below nominal) will result in a signal a thousand times weaker than nominal (at CESR, such a factor is enough to lower the signal down to the observed background rate). At the NLC there will be an extensive initial phase of machine development, with weak, relatively broad beams. CB provides the large enhancement needed to observe precisely such weak beams. This is a first, important property of coherent beamstrahlung.

The relativistically invariant coherence condition is

$$\int d^4 \rho(x) e^{ik \cdot x} \sim 1,$$

where $k$ is the observed photon 4-momentum, and $\rho(x)$ is the electron probability distribution in space-time (normalized to 1). Given that both the beam and the emitted photons are extremely longitudinal, the coherence condition becomes simply

$$\frac{\lambda}{\sigma_z} \sim 1.$$

If the wavelength is of order of the beam length, the radiation emitted will be coherent, otherwise it will be incoherent. At CESR, this translates to wavelengths greater than 1cm. At the NLC, the wavelengths of interest are those in excess of 0.1mm. The expected enhancement is also huge, of order $N_{1,2} \sim 10^{10}$. A second important property of CB is that one can reasonably expect it to be background-free. The SR from the magnets will be incoherent, and therefore much weaker, because the magnets are much longer than $\lambda$.

It becomes immediately clear that at CESR observation is hampered by having a beam pipe whose diameter is comparable to the wavelength (3cm), resulting in the well-known absorption of EM waves as they travel down the beam pipe[4]. At the NLC, however, 0.1 mm waves are a factor of 25 shorter than the beam pipe diameter and will be able to travel long distances. Detection of signals is also much easier in the 0.1 mm range than in the 1cm range. A third important property is that at the NLC the experimental conditions are much more favorable than at current storage rings (basically due to much shorter beams).

Another feature of coherent beamstrahlung radiation can be inferred directly from Fig. 2. It is clear that, for radiation to become coherent, the whole beam has to move in a certain transverse direction coherently. In Figs. 2c) and 2d), different parts of the beams move in opposite directions and interfere destructively. It is only in the case of a beam-beam offset (Fig. 2b) that the beam as a whole moves vertically. Therefore coherent radiation will only appear in the presence of a non-zero offset and will primarily measure an offset. It is also clearly evident that (as long as the offset along the $x-$axis is not significant) only the $y-$component of the polarization will be coherent, as the coherent motion is purely along that direction. Thus coherent radiation will isolate and amplify two single components (one for each beam) of the diagrams of Fig. 3.

To produce quantitative results, the beam-beam simulation program of Ref.[2] was developed further to include coherent radiation. This program is one of many cloud-in-cell programs existing on the market, and since it was developed for CESR, beamstrahlung energy loss by the beam particles is not included. This is a small deficiency of the program that does not affect the main results produced below - at the NLC (Table 1) the typical beamstrahlung loss is of order a few to several percent (small corrections like these can be introduced at a later stage).

In this, and all other, programs, radiation is calculated for each interaction of one cell of beam 1 with each cell of beam 2. The radiation is added together to provide a final result. When adding incoherent beamstrahlung, under the assumption discussed above that one can recover 100% linear polarization, one makes use of the following formulae[2] to find the force exerted by all cells (index $i$) in beam 2 on a cell in the beam 1 (index $j$) is[2]

$$\Delta r''_{ij} = -\frac{2N_f r_e^2}{\gamma} \sum \frac{P_{2i}}{b_{ij}^2},$$

$$F_{ij} = \frac{\gamma m c^2}{2\Delta z} \Delta r''_{ij},$$

$\gamma$ is the relativistic factor, $m$ the electron mass, $c$ the speed of light, $\Delta z$ the step along the beam collision axis, and $\Delta r''$ the (transverse) deflection during such a step. $F_{ij}$ is the force exerted on one particle of beam 1 by the whole beam 2.

The $P$ are the fractional charge population in each cell, and $b$ is the transverse impact parameter between the centers of the two cells. The energy vector $U_1$ (which is simply the two polarization components of IB) for beam 1 is computed by summing[2]

$$U_{ix} = \sum \Delta U_{ix j} = \frac{2N_f r_e \Delta r_{x j}^2}{3 m c^2} \sum P_{ij} F_{ij x}^2,$$

$$U_{iy} = \sum \Delta U_{iy j} = \frac{2N_f r_e \Delta r_{y j}^2}{3 m c^2} \sum P_{ij} F_{ij y}^2.$$
become

\[
W_{1x} = \frac{2N^2 r_e \Delta x \gamma^2}{3mc^2} \left( \sum P_j e^{ik_x x} F_{1x} \right)^2, \quad (9)
\]

\[
W_{1y} = \frac{2N^2 r_e \Delta y \gamma^2}{3mc^2} \left( \sum P_j e^{ik_y y} F_{1y} \right)^2, \quad (10)
\]

and we apply the normalization condition \( W_0 = U_0 \). The limitation of the method is that there exists a transition region between incoherent and coherent bremsstrahlung, where the program will not work. The two sets of formulae do coincide, in the limit of very large statistics and short wavelength, and in a way that is consistent with Ref.[10]. However the cell population \( C \) inside each beam is finite (typically \( C \sim 3 \times 10^8 \)), and we found that the CB program would be numerically stable only if the coherent enhancement was greater than the number of cells

\[
\frac{W}{W_0} > C.
\]

![Graph showing CB yield as a function of beam-beam offset](image1)

**FIG. 4:** CB yield as a function of the beam-beam offset. The simulations were done with NLC nominal conditions (Table I), but weaker beams \((N_1 = N_2 = 0.3 \times 10^{10}, \sigma_{y_1} = \sigma_{y_2} = 19\mu m)\). Plots are shown for four different wavelength-beam length ratios. The LINX working points are discussed in the main text. The markers locate the points where the simulation was performed.

The total power emitted in the microwave region may exceed 10W when full strength NLC beams are offset by a few \( \sigma_y \) (Table I). The main simulation results are shown in Figs. 4-6 for “weak” beams. In Fig. 4, the microwave power (in units of IB power) is shown as a function of the beam-beam vertical offset (in beam width units). The curves show the dependence of the coherent yield for various \( R \) ratios.

![Graph showing CB ratio of yields (beam 1 versus beam 2) as a function of beam-beam offset](image2)

**FIG. 5:** CB ratio of yields (beam 1 versus beam 2) as a function of the beam-beam offset. The simulations conditions are described in Fig. 4, but \( \sigma_{x_2} = 88\mu m \).
impossible. As a working example, consider the LINX facility[11], which would provide a major proof of principle for future linear colliders. LINX would produce and collide 50 nm-wide beams, and use them to measure the beam jitter. A simple way to do that is depicted in Fig. 4. The beams are brought to a collision, then displaced by a quantity of order 0.5σy. From Fig. 4, one can see that a 0.1 nm jitter will produce emitted power fluctuations of order 10%. Moving the beams to a separation of 3.0σy would provide good online calibration against possible instrumental jitter.

IV. CONCLUSIONS.

At the NLC, incoherent beamstrahlung (IB) should retain most of its usefulness as a near-complete BBC monitor. The expected light signals are large, and there are, on paper, methods to reduce machine backgrounds by several orders of magnitude. Experimental issues will be discussed in a future paper. Once the visible SR backgrounds calculations at the IP will be available, one or more of the four background rejection methods will be implemented in a final design.

At the NLC, there will be a slow machine drift that incoherent beamstrahlung (IB) can monitor almost completely by itself (six out of seven dof, with the seventh one being measurable by scanning beams), exactly like at CESR.

At the NLC, there will be also substantial beam jitter (also potentially a 7-dimensional phenomenon). IB will be able to monitor only four of these seven dof. In part to counter this limitation, we have introduced the idea of measuring the coherent, microwave part of beamstrahlung. This part of the spectrum provides two extra, independent dof, bringing the total back to six and effectively providing almost complete monitoring. CB will also provide precision measurements of the beam length, and will work initially with very weak beams.

Like the beamstrahlung diagrams of Fig. 3, the CB plots presented here are semi-universal. What that means is that, up to small corrections related to the beams disruption during the BBC, Figs. (4-6) are universal. That is why both axes are scaled variables, with each curve depending on a third, scaled parameter.

In the case of CB, the rates are enormous (Table I), which allows the usage of anything sensitive to microwaves (including microantennas inside the beam pipe). CB will probably be measured above a certain threshold in the beam-beam offset, of order 0.1σy. We note that the experimental issues related to CB are entirely technical, including how to avoid burning the microwave detector, how to ensure a very large dynamic range, and how to read the microwave signal for each bunch (with a time separation of 1.4 nsec).

Finally, one needs to point out that there are some significant differences between the low energy beamstrahlung method and the beam-beam deflection method (see for example[12]). Beamstrahlung is purely passive, and it is sensitive, in a passive way, to pathologies other than offsets. It also measures the ratio of the two σy in a purely passive way, so that one beam's detuning is diagnosed instantly. When the beams are scanned through one another, this method measures both σy (as opposed to the quadratic sum of the two). This method will not be affected by different bunch lengths, and will provide a positive signal when the beams are colliding properly, whereas the beam-beam deflection provides zero signal. Finally, beamstrahlung is sensitive to vertical jitter far smaller than what the beam-beam deflection method can measure.

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